Use an appropriate linear approximation to estimate $\cos^{-1} 0.05$.

SCORE: _____ / 5 PTS

$f(x) = \cos^{-1} x$ NEAR $x = 0$

$f'(x) = -\dfrac{1}{\sqrt{1-x^2}}$

1

$f(x) \approx f(0) + f'(0)(x-0)$

$= \cos^{-1} 0 - \dfrac{1}{\sqrt{1-0^2}} x$

$= \dfrac{\pi}{2} - x$

$\cos^{-1} 0.05 \approx \dfrac{\pi}{2} - \dfrac{1}{20}$

1 $1\frac{1}{2}$ $1\frac{1}{2}$

The angle between two adjacent sides of a triangle is 120°. **SCORE: _____ / 8 PTS**

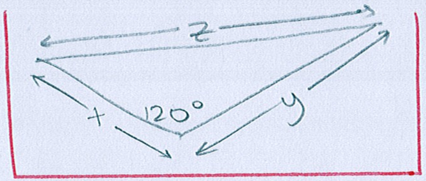
One adjacent side is 6 inches long and shrinking by an inch every minute.

The other adjacent side is 1 inch long and expanding by $1\frac{1}{2}$ inches every minute.

At that instant, is the third side expanding or shrinking, and how quickly ?

<u>You must state/show clearly what each variable you use represents.</u>

<u>You must show the units during the intermediate steps of your work, and you must state the units for the final answer.</u>



1

$$\left.\frac{dx}{dt}\right|_{x=6\,IN} = -1\ \frac{IN}{MIN} \qquad \left.\frac{dy}{dt}\right|_{y=1\,IN} = \frac{3}{2}\ \frac{IN}{MIN}$$

WANT $\left.\frac{dz}{dt}\right|_{x=6\,IN,\ y=1\,IN}$

$$\boxed{z^2 = x^2 + y^2 - 2xy\cos 120^\circ = x^2 + y^2 + xy}$$ 1

WHEN $x = 6\,IN$, $y = 1\,IN$

$$z^2 = (6\,IN)^2 + (1\,IN)^2 + (6\,IN)(1\,IN)$$

$$z^2 = 43\,IN^2$$

$$z = \sqrt{43}\,IN$$

$$\boxed{2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + \frac{dx}{dt}y + x\frac{dy}{dt}}$$ 1

$$\boxed{2\sqrt{43}\,IN\ \frac{dz}{dt} = 2(6\,IN)\left(-1\frac{IN}{MIN}\right) + 2(1\,IN)\left(\frac{3}{2}\frac{IN}{MIN}\right) + \left(-1\frac{IN}{MIN}\right)(1\,IN) + (6\,IN)\left(\frac{3}{2}\frac{IN}{MIN}\right)}$$ 2

$$2\sqrt{43}\ \frac{dz}{dt} = (-12 + 3 - 1 + 9)\frac{IN}{MIN} = -1\ \frac{IN}{MIN}$$

$$\frac{dz}{dt} = -\frac{1}{2\sqrt{43}}\ \frac{IN}{MIN}$$

1 1

THE THIRD SIDE IS $\boxed{\text{SHRINKING}}$ BY $\boxed{\frac{1}{2\sqrt{43}}}$ $\boxed{\text{INCHES PER MINUTE}}$

1 ONLY IF UNITS SHOWN IN EQUATION

Prove the derivative of $\operatorname{sech} x$ using the known derivative of $\cosh x$, along with the quotient rule. SCORE: ____ / 4 PTS

Show all work. You must NOT use the chain rule.

$$\frac{d}{dx}\operatorname{sech} x = \frac{d}{dx}\frac{1}{\cosh x}$$

$$= \frac{0 \cdot \cosh x - 1 \cdot \sinh x}{\cosh^2 x} \quad 2 \quad \tfrac{1}{2}$$

$$= \boxed{\frac{-\sinh x}{\cosh^2 x}} \; 1 \; = \boxed{-\operatorname{sech} x \tanh x} \; \tfrac{1}{2}$$

MUST HAVE NEGATIVE IN FRONT

A ferris wheel with a radius of 10 meters is rotating at a constant rate. **SCORE: _____ / 13 PTS**

The bottom of the wheel is 1 meter above the ground.

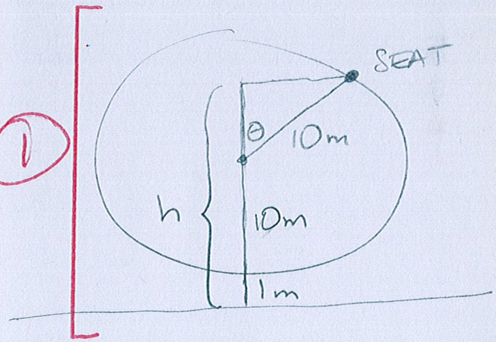
A certain seat on the wheel is currently 17 meters above the ground, and rising at $\frac{1}{2}$ meter per second.

How quickly is the ferris wheel turning ?

<u>You must state/show clearly what each variable you use represents.</u>

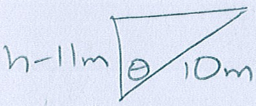
<u>You must show the units during the intermediate steps of your work, and you must state the units for the final answer.</u>

1

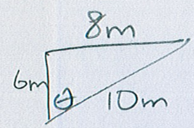


$$\frac{dh}{dt} = \frac{1}{2}\frac{m}{s}$$

WANT $\left.\frac{d\theta}{dt}\right|_{h=17m}$



WHEN $h = 17m$



$$\sin\theta = \frac{8m}{10m} = \frac{4}{5}$$

$$\boxed{\cos\theta = \frac{h - 11m}{10\ m}} \quad 4$$

$$10\cos\theta\ m = h - 11m$$

$$\boxed{-10\sin\theta\frac{d\theta}{dt}\ m = \frac{dh}{dt}} \quad 2$$

$$\boxed{-\cancel{10}^{2} \cdot \boxed{\frac{4}{\cancel{5}}}\ 2\ \frac{d\theta}{dt}\ \cancel{m} = \frac{1}{2}\frac{\cancel{m}}{s}}$$

2

$$\frac{d\theta}{dt} = -\frac{1}{16 s} \text{ OR } -\frac{1}{16} \text{ RADIANS PER SECOND}$$

THE WHEEL IS TURNING AT $\boxed{\frac{1}{16}}$ $\boxed{\text{RADIANS PER SECOND}}$

1   1 ONLY IF UNITS SHOWN IN EQUATION